

J. P. B. C. de Melo · T. Frederico

# Eletroweak Form Factors in the Light-Front for Spin-1 Particles <sup>★</sup>

Version of February 1, 2012

**Abstract** The contribution of the light-front valence wave function to the electromagnetic current of spin-1 composite particles is not enough to warranty the proper transformation of the current under rotations. The naive derivation of the plus component of the current in the Drell-Yan-West frame within an analytical and covariant model of the vertex leads to the violation of the rotational symmetry. Computing the form-factors in a quasi Drell-Yan-West frame  $q^+ \rightarrow 0$ , we were able to separate out in an analytical form the contributions from Z-diagrams or zero modes using the instant-form cartesian polarization basis.

**Keywords** Spin-1 Particles · Electromagnetic Current · Electromagnetic Form factors · Light-Front Field Theory

## 1 Introduction

Light-front models are useful to describe hadronic bound states, like mesons or baryons due to its particular boost properties [1; 3]. However, the light-front description in a truncated Fock-space breaks the rotational symmetry because the associated transformation is a dynamical boost [4; 5; 6; 7]. Therefore, an analysis with covariant analytical models, can be useful to pin down the main missing features in a truncated light-front Fock-space description of the composite system. In this respect, the rotational symmetry breaking of the plus component of the electromagnetic current, in the Drell-Yan frame, was recently studied within an analytical model for the spin-1 vertex of a composite two-fermion bound state [4; 5].

It was shown that, if pair term contributions are ignored in the evaluation of the matrix elements of the electromagnetic current, the covariance of the form factors is lost [4; 5; 6; 9; 10].

The complete restoration of covariance in the form factor calculation is found only when pair terms or zero modes contributions to the matrix elements of the current are considered [5; 6; 10; 11; 12].

The extraction of the electromagnetic form-factors of a spin-1 composite particle from the microscopic matrix elements of the plus component of the current ( $J^+ = J^0 + J^3$ ) in the Drell-Yan frame (momentum transfer  $q^+ = q^0 + q^3 = 0$ ), based only on the valence component of the wave function, presents ambiguities due to the lacking of the rotational invariance of the current model [13; 14]. In the Breit-frame with momentum transfers along the transverse direction (the Drell-Yan condition is

---

<sup>★</sup> Presented by J. P. B. C. de Melo at LIGHTCONE 2011, 23 -27 May, 2011 Dallas.

J. P. B. C. de Melo  
Laboratório de Física Teórica e Computação Científica, Universidade Cruzeiro do Sul, 01506-000, São Paulo, Brazil.

T. Frederico  
Departamento de Física, Instituto Tecnológico de Aeronáutica, CTA, 12228-900, São José dos Campos, Brazil.

satisfied) the current  $J^+$  has four independent matrix elements, although only three form factors exist. Therefore, the matrix elements satisfies an identity, known as the angular condition [13], which is violated.

Several extraction schemes for evaluating the form-factors were proposed, and in particular we consider the suggestion made in Ref. [13]. It was found in a numerical calculation of the  $\rho$ -meson electromagnetic form factors considering only the valence contribution [4], that the prescription proposed by [13] to evaluate the form-factors, produced results in agreement with the covariant calculations. In Ref. [4], it was used an analytical form of the  $\rho$ -quark-antiquark vertex. Later, in Ref. [9], it was shown that the above prescription eliminates the pair diagram contributions to the form factors, using a simplified form of the model, when the matrix elements of the current were evaluated for spin-1 light-cone polarization states. This nice result was thought to be due to the use of the particular light-cone polarization states. Here, we will show in a straightforward and analytic manner that the cancelation of the pair contribution in the evaluation of the form factors using the prescription from Ref. [13] also happens for the instant form polarization states in the cartesian representation, generalizing the previous conclusion [9]. Our aim, is to expose in a simple and detailed form, how the pair terms appear in the matrix elements of the current evaluated between instant form polarization states, and their cancelation in the form factors using the correct prescription. Therefore, we conclude that this property is more general than realized before. But, in the case of spin-0 composite particles (like the pion) with the correspondent form of the analytical model, the plus component of the electromagnetic current in the Breit-frame, with  $q^+ = 0$ , does not have contributions from pair terms [12; 15].

## 2 Light-Front Model Spin-1 Particles and Wave Function

The electromagnetic current has the following general form for spin-1 particles [18]:

$$J_{\alpha\beta}^\mu = [F_1(q^2)g_{\alpha\beta} - F_2(q^2)\frac{q_\alpha q_\beta}{2m_v^2}](p^\mu + p'^\mu) - F_3(q^2)(q_\alpha g_\beta^\mu - q_\beta g_\alpha^\mu), \quad (1)$$

where  $m_v$  is the mass of the vector particle,  $q^\mu$  is the momentum transfer,  $p^\mu$  and  $p'^\mu$  is on-shell initial and final momentum respectively. From the covariant form factors  $F_1$ ,  $F_2$  and  $F_3$ , one can obtain the charge ( $G_0$ ), magnetic ( $G_1$ ) and quadrupole ( $G_2$ ) form factors (see e.g. [4]).

The matrix elements of the electromagnetic current  $\mathcal{J}_{ji} = \epsilon_j'^\alpha \epsilon_i^\beta J_{\alpha\beta}^\mu$  in the impulse approximation are written as [4]:

$$\mathcal{J}_{ji}^+ = i \int \frac{d^4 k}{(2\pi)^4} \frac{Tr[\epsilon_j'^\nu \Gamma_\beta(k, k - p_f)(\not{k} - \not{p}_f + m)\gamma^+(\not{k} - \not{p}_i + m)\epsilon_i^\mu \Gamma_\alpha(k, k - p_i)(\not{k} + m)]\Lambda(k, p_f)\Lambda(k, p_i)}{(k^2 - m^2 + i\epsilon)((k - p_i)^2 - m^2 + i\epsilon)((k - p_f)^2 - m^2 + i\epsilon)} \quad (2)$$

where  $\epsilon_j'$  and  $\epsilon_i$  are the polarization four-vectors of the final and initial states, respectively and  $m$  is the quark mass. The electromagnetic form-factors are calculated in the Breit frame with the Drell-Yan-West condition, which gives the momentum transfer  $q^\mu = (0, q_x, 0, 0)$ , the particle initial momentum  $p^\mu = (p^0, -q_x/2, 0, 0)$  and the final one  $p'^\mu = (p^0, q_x/2, 0, 0)$ . We use  $\eta = -q^2/4m_v$  and  $p^0 = m_v\sqrt{1+\eta}$ . The polarization four-vectors in instant-form basis are given by  $\epsilon_x^\mu = (-\sqrt{\eta}, \sqrt{1+\eta}, 0, 0)$ ,  $\epsilon_y^\mu = (0, 0, 1, 0)$ ,  $\epsilon_z^\mu = (0, 0, 0, 1)$ , for the initial state and by,  $\epsilon_x'^\mu = (\sqrt{\eta}, \sqrt{1+\eta}, 0, 0)$ ,  $\epsilon_y'^\mu = \epsilon_y^\mu$ ,  $\epsilon_z'^\mu = \epsilon_z^\mu$ , for the final state. The regularization function  $\Lambda(k, p_{i(f)}) = N/((p - k)^2 - m_R + i\epsilon)^2$  is enough to render finite the photo-absorption amplitude; the regularization parameter is  $m_R$ . The vertex function  $m_v - q\bar{q}$  for the vector particles utilized (see ref. [4] for details) is given below:

$$\Gamma^\mu(k, p) = \gamma^\mu - \frac{m_v}{2} \frac{2k^\mu - p^\mu}{p \cdot k + m_v m - i\epsilon}. \quad (3)$$

The vector particle is on-mass shell;  $m_v$  is the vector bound state mass, and its four momentum  $p^\mu = k^\mu - k'^\mu$ . After the integration in the light-front energy ( $k^- = (\mathbf{k}_\perp^2 + m^2)/k^+$ ), the light-front wave function is writing like:

$$\Phi_i(x, \mathbf{k}_\perp) = \frac{N^2}{(1-x)^2(m_v^2 - M_0^2)(m_v^2 - M_R^2)^2} \epsilon_i \cdot [\gamma - \frac{\mathbf{k}}{\frac{M_0}{2} + m}]. \quad (4)$$

The extraction of the electromagnetic form factor with the plus component of the electromagnetic current and the angular condition are discussed next section.

### 3 Light-Front Spin Basis and the Angular Condition

The matrix elements of the electromagnetic current expressed in terms of the current in the instant form spin basis, after the use of the Melosh rotation [4; 19; 20] is:

$$\begin{aligned} I_{11}^+ &= \frac{J_{xx}^+ + (1+\eta)J_{yy}^+ - \eta J_{zz}^+ - 2\sqrt{\eta}J_{zx}^+}{2(1+\eta)}, \quad I_{10}^+ = \frac{\sqrt{2\eta}J_{xx}^+ + \sqrt{2\eta}J_{zz}^+ - \sqrt{2}(\eta-1)J_{zx}^+}{2(1+\eta)} \\ I_{1-1}^+ &= \frac{-J_{xx}^+ + (1+\eta)J_{yy}^+ + \eta J_{zz}^+ + 2\sqrt{\eta}J_{zx}^+}{2(1+\eta)}, \quad I_{00}^+ = \frac{-\eta J_{xx}^+ + J_{zz}^+ - 2\sqrt{\eta}J_{zx}^+}{(1+\eta)}, \end{aligned} \quad (5)$$

here the matrix elements of the electromagnetic current are calculated with the plus component of the current, "+" and the light-front polarization states denoted as  $I_{m'm}^+$ .

In the Breit-frame ( $q^+ = 0$ ), the angular condition is translated by the equation [4; 19]:

$$\Delta(q^2) = (1+2\eta)I_{11}^+ + I_{1-1}^+ - \sqrt{8\eta}I_{10}^+ - I_{00}^+ = (1+\eta)(J_{yy}^+ - J_{zz}^+) = 0. \quad (6)$$

In the case of the instant form spin basis, the angular condition is  $J_{yy}^+ = J_{zz}^+$  [20]. The prescription adopted by Grach and Kondratyuk [13] eliminate the matrix elements  $I_{00}^+$  in order to compute the electromagnetic form factors for spin-1 particles.

The terms of the trace calculated with only the  $\gamma^\mu$  structure of the vertex, of the Eq.(3), is writing below:

$$Tr[gg]_{ji} = Tr[\not{\epsilon}_j^\alpha(\not{k} - \not{p}' + m)\gamma^+(\not{k} - \not{p} + m)\not{\epsilon}_i^\alpha(\not{k} + m)] . \quad (7)$$

The trace is calculated with the light-front coordinates, ( $k^+ = k^0 + k^3$ ,  $k^- = k^0 - k^3$ ,  $k_\perp = (k_x, k_y)$ ) and the  $k^-$  dependence is separate, (in the Eq.(7)), then, we get the results below:

$$Tr[gg]_{ji}^{+Z} = \frac{k^-}{2} Tr[\not{\epsilon}_j^\alpha(\not{k} - \not{p}' + m)\gamma^+(\not{k} - \not{p} + m)\not{\epsilon}_i^\alpha\gamma^+] . \quad (8)$$

For the polarization four-vectors ( $\epsilon_i^\mu, \epsilon_j^\mu$ ), the traces above are given by:

$$\begin{aligned} Tr[gg]_{xx}^{+Z} &= -k^- \frac{\eta}{8} R_{gg}, \quad Tr[gg]_{yy}^{+Z} = 4k^-(p^+ - k^+)^2, \\ Tr[gg]_{zz}^{+Z} &= \frac{1}{8} k^- R_{gg}, \quad Tr[gg]_{zx}^{+Z} = -k^- \frac{\sqrt{\eta}}{8} R_{gg}, \end{aligned} \quad (9)$$

where  $R_{gg} = 4 Tr[(\not{k} - \not{p}' + m)\gamma^+(\not{k} - \not{p} + m)\gamma^-]$ . The trace equation, Eq.(9), yield the following relations:

$$\begin{aligned} Tr[gg]_{xx}^{+Z} &= -\eta Tr[gg]_{zz}^{+Z}, \\ Tr[gg]_{zx}^{+Z} &= -\sqrt{\eta} Tr[gg]_{zz}^{+Z}. \end{aligned} \quad (10)$$

The zero-modes contributions for the direct term coupling given by:

$$J_{ji}^{+Z}[gg] = \lim_{\delta^+ \rightarrow 0^+} \sum_{r=4,5; s=4,6} \int [d^4k]^{+Z} \frac{Tr[gg]_{ji}^{+Z}}{[1][2][3][r]^2[s]^2}, \quad (11)$$

where  $[d^4k]^Z = [d^4k] \theta(p'^+ - k^+) \theta(k^+ - p^+)$ , and the denominators are given by:  $[1] = [k^2 - m^2 + i\epsilon]$ ,  $[2] = [(k-p)^2 - m^2 + i\epsilon]$ ,  $[3] = [(k-p')^2 - m^2 + i\epsilon]$ ,  $[4] = [(k-p)^2 - m_R^2 + i\epsilon]$ ,  $[5] = [(k-p')^2 - m_R^2 + i\epsilon]$ ,  $[6] = [k^2 - m_R^2 + i\epsilon]$  and  $p'^+ = p^+ + \delta^+$ . The integration of the Eq.(11) is performed with the *pole dislocation method*, developed in the reference [11], after that, the contribution of the *zero modes* for the matrix element  $J_{yy}^{+Z}$  of the electromagnetic current is zero and the matrix element  $J_{yy}^+$  of the electromagnetic current not have a pair term contribution.

Using the prescription of the Ref.[13], the electromagnetic form factors for spin-1 particles are writing with matrix elements of the electromagnetic current in instant form spin basis as:

$$\begin{aligned} G_0^{GK} &= \frac{1}{3} (J_{xx}^+ + \eta J_{zz}^+ + (2-\eta)J_{yy}^+), \quad G_1^{GK} = J_{yy}^+ - \frac{1}{\sqrt{\eta}} (J_{zx}^+ + \sqrt{\eta}J_{zz}^+), \\ G_2^{GK} &= \frac{\sqrt{2}}{3} (J_{xx}^+ + \eta J_{zz}^+ - (1+\eta)J_{yy}^+), \end{aligned} \quad (12)$$

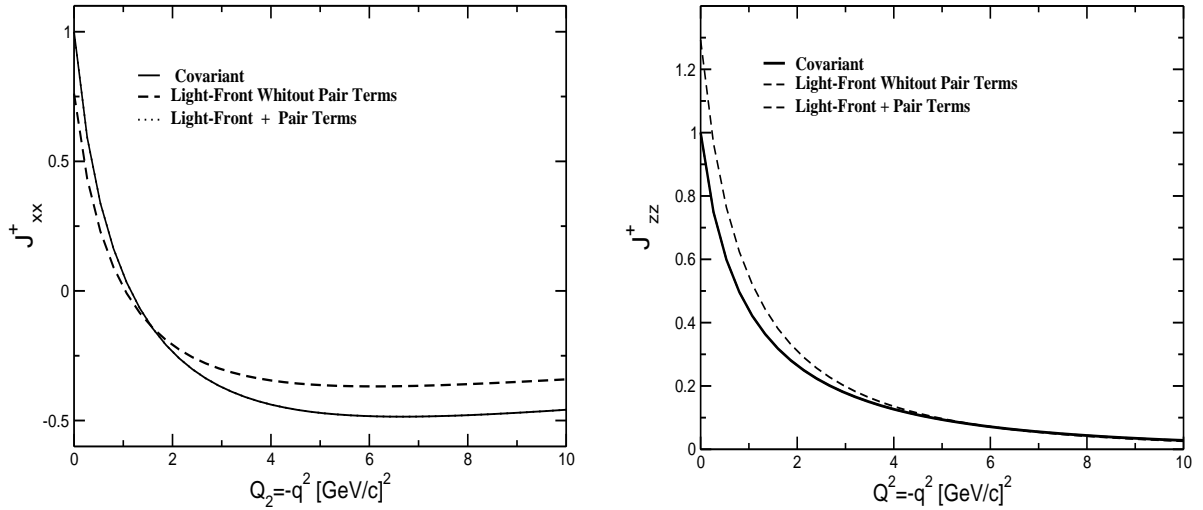
here the *superscripts GK* in the equation above is the initials for the authors of the Ref. [13], Grach, I. and Kondratyuk, L. A.

Considering the relations in Eq.(10), the prescription given by Grach [13], cancel out the zero-modes contribution to the electromagnetic form factors for this part of the vertex function (see the figures 1 and 2). Then the angular condition in the Cartesian spin basis with the zero-modes contribution is [4]:

$$\Delta(q^2) = J_{yy}^+ - J_{zz}^+ = J_{yy}^{+val} - J_{zz}^{+val} - J_{zz}^{+Z} = 0, \quad (13)$$

because the zero modes contribution to the matrix elements  $J_{yy}^{+Z}$  of the electromagnetic current is zero; the angular condition is only due to the valence components of the electromagnetic current,  $J_{yy}^{+val}$ ,  $J_{zz}^{+val}$ , and to the zero modes contribution to the electromagnetic current  $J_{zz}^{+Z}$ .

The figures 1 and 2 show the results for the electromagnetic matrix elements of the current calculated with the instant form and the light-front approach. After the zero-modes inclusion, the covariance are restore.



**Fig. 1** The plots show the matrix elements of the electromagnetic current,  $J_{xx}^+$  and  $J_{zz}^+$ , calculated with the coupling  $(\gamma^\mu, \gamma^\nu)$  in the light-front approach and the instant form calculation; the parameters utilized are  $m = 0.430 \text{ GeV}$ ,  $m_v = 0.770 \text{ GeV}$  and the regulator mass as  $m_R = 1.8 \text{ GeV}$ .

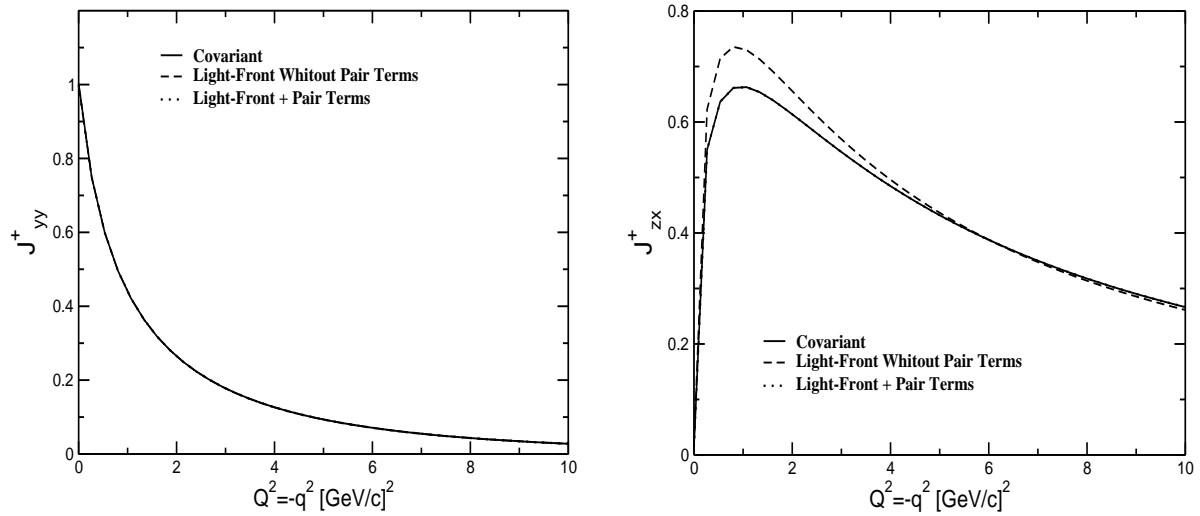
## 4 Conclusions

Following the Ref. [4] we separate the structure of the vertex Eq.(3) in the valence contribution and the zero-modes for the spin-1 particles.

The prescription suggested by the Ref. [13] utilized in the calculation of the electromagnetic form factor for spin-1 particles cancel out the zero-modes contribution to the matrix elements for the electromagnetic current; then the electromagnetic form factors for spin-1 calculated with the *Grach* and *Kondratyuk* [13] are free of the non-valence contribution and only the valence part of the electromagnetic current contributed to the electromagnetic form factors.

It is possible to generalize the conclusion found here for the full electromagnetic matrix elements of the spin-1 particles with the vertex function, Eq.(3); i.e, the zero-modes not contributed to the electromagnetic form factors if the prescription of the Ref.[13] it is adopted.

**Acknowledgments.** This work was supported in part by the Brazilian agencies FAPESP (Fundação de Amparo à Pesquisa do Estado de São Paulo) and CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico). J. P. B. C. de Melo thanks the organizer of the Light-Cone 2011 for the invitation.



**Fig. 2** The plots show the matrix elements of the electromagnetic current,  $J_{yy}^+$  and  $J_{zx}^+$ , calculated with the coupling  $(\gamma^\mu, \gamma^\nu)$  in the light-front approach and the instant form calculation; the parameters utilized are  $m = 0.430 \text{ GeV}$ ,  $m_v = 0.770 \text{ GeV}$  and the regulator mass as  $m_R = 1.8 \text{ GeV}$ .

## References

1. M.V. Terent'ev: *On the structure of wave functions of mesons as bound states of relativistic quarks.* (1980) Sov. J. Nucl. Phys. **24**, 106 (1976).
2. L.A. Kondratyuk and M.V. Terent'ev: *The Scattering Problem For Relativistic Systems With Fixed Number Of Particles,* Sov. J. Nucl. Phys. **31**, 561 (1980).
3. S. J. Brodsky, H.-C. Pauli, and S. S. Pinsky: Quantum chromodynamics and other field theories on the light cone. Phys. Rep. **301**, 299 (1998).
4. J.P.B.C. de Melo and T. Frederico: *Covariant and light front approaches to the rho meson electromagnetic form-factors.* Phys. Rev. C **55**, 2043 (1997).
5. J.P.B.C. de Melo, J.H.O.Sales, T. Frederico, and P.U. Sauer: *Pairs in the light front and covariance.* Nucl. Phys. **A631**, 574c (1998).
6. J.P.B.C. de Melo, T. Frederico, H.W.L. Naus, and P.U. Sauer: *Pion electromagnetic current in the light cone formalism.* Nucl. Phys. **A660**, 219 (1999).
7. J. H. O. Sales, T. Frederico, B. V. Carlson and P. U. Sauer: *Light front Bethe-Salpeter equation.* Phys. Rev. **C61**, 044003 (2000).
8. J. H. O. Sales, T. Frederico, B. V. Carlson and P. U. Sauer: *Renormalization of the ladder light front Bethe-Salpeter equation in the Yukawa model.* Phys. Rev. **C63**, 064003(2001).
9. B.L.G. Bakker and C.R. Ji: *The Vector meson form-factor analysis in light front dynamics.* Phys. Rev. **D65**, 116001 (2002).
10. Ho-Meoyng Choi and C. R. Ji: *Electromagnetic structure of the rho meson in the light front quark model.* Phys. Rev. **D70**, 053015 (2004).
11. H.W.L. Naus, J.P.B.C. de Melo, and T. Frederico: *Ward-Takahashi identity on the light front.* Few-Body Systems **24**, 99 (1998).
12. J.P.B.C. de Melo, H.W. L. Naus, and T. Frederico: *Pion electromagnetic current in the light cone formalism.* Phys. Rev. **C59**, 2278 (1999).
13. I.L.Grach and L.A.Kondratyuk: *Electromagnetic Form-factor Of Deuteron In Relativistic Dynamics. Two Nucleon And Six Quark Components.* Sov. J. Nucl. Phys. **39**, 198 (1984).
14. I.L.Grach, L.A. Kondratyuk, and M.Strikman: *Is the structure In the Deuteron magnetic form-factor at  $Q^2$  approximately  $= 2 - \text{GeV}^2$  a new evidence for nuclear core?.* Phys. Rev. Lett. **62**, 387 (1989).
15. J. .P. B. C. de Melo, T. Frederico, E. Pace, and G. Salmé: *Pair term in the electromagnetic current within the front form dynamics: Spin-0 case.* Nucl. Phys. A **07**, 399 (2002).
16. J. .P. B. C. de Melo, T. Frederico, E. Pace, and G. Salmé: *Frame dependence of the pair contribution to the pion electromagnetic form-factor in a light front approach.* Braz. Jour. of Phys. **33**, 301 (2003).
17. L.L. Frankfurt and M. Strikman: *Relativistic Description of the Deuteron: Elastic and Inelastic Form-Factors in the Region Kinematically Forbidden for Scattering on a Free Nucleon.,* Nucl. Phys. **B148**, 107 (1979).
18. L.L. Frankfurt and M. Strikman: *High-Energy Phenomena, Short Range Nuclear Structure and QCD.* Phys. Rep. **76**, 215 (1981).

- 
19. B. D. Keister and W. N. Polizou: *Relativistic Hamiltonian dynamics in nuclear and particle physics*. Adv. Nucl. Phys. **20**, 225 (1991).
  20. L. L. Frankfurt, T. Frederico, and M. Strikman: *Deuteron form-factors in the light cone quantum mechanics 'good' component approach*. Phy. Rev. **C48**, 2182 (1993).